TRINITY COLLEGE

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Year 11 Mathematics Specialist
Test 2/3 2020

Section 1 Calculator Free

Component Vectors & Geometric Proof

STUDENT'S NAME Solutions (PRESSER)

DATE: Wednesday 13 May **TIME:** 25 minutes **MARKS:** 26

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Given that $\mathbf{a} = -3\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{c} = 3\mathbf{i} - 2\mathbf{j}$ determine:

(a) a unit vector in the same direction as $\mathbf{b}$ [2]

$$\hat{\underset{\sim}{b}} = \frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\end{pmatrix}$$

(b) $|\mathbf{a}+\mathbf{b}+\mathbf{c}| = \left|\begin{pmatrix}2\\3\end{pmatrix}\right|$ [2]

$$= \sqrt{13}$$

(c) a vector that is parallel to $\mathbf{a}+\mathbf{b}+\mathbf{c}$ with a magnitude of 4. [2]

$$\frac{4}{\sqrt{13}}\begin{pmatrix}2\\3\end{pmatrix}$$

2. (4 marks)

The unit vector $\mathbf{u} = a\mathbf{i} - b\mathbf{j}$ is perpendicular to $4\mathbf{i} + 3\mathbf{j}$. If $a > 0$, determine the value of a and b

$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 0$$

$$\Rightarrow 4a + 3b = 0$$

$$\Rightarrow b = \frac{4a}{3}$$

$$|\underset{\sim}{u}| = 1$$

$$\Rightarrow a^2 + b^2 = 1$$

$$\Rightarrow a^2 + \left(\frac{4a}{3}\right)^2 = 1$$

$$\Rightarrow a^2 + \frac{16a^2}{9} = 1$$

$$\Rightarrow 25a^2 = 9$$

$$\Rightarrow a = \pm\frac{3}{5}$$

but $a > 0$

$$\therefore a = \frac{3}{5}$$

$$b = \frac{4}{3} \cdot \frac{3}{5}$$

$$b = \frac{4}{5}$$

3. (4 marks)

Consider the following statement:

If ABCD is a parallelogram, then △ABD and △CBD are congruent.

(a) Determine the converse statement of this premise [1]

If △ABD and △CBD are congruent, then ABCD is a parallelogram

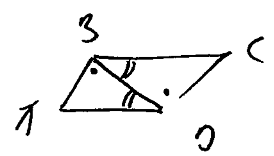
(b) (i) Determine the contrapositive statement of this premise. [1]

If △ABD and △CBD are not congruent, then ABCD is not a parallelogram

(ii) Is the contrapositive statement true or false? Explain. [2]

The original statement is true

Therefore the contrapositive statement is also true.

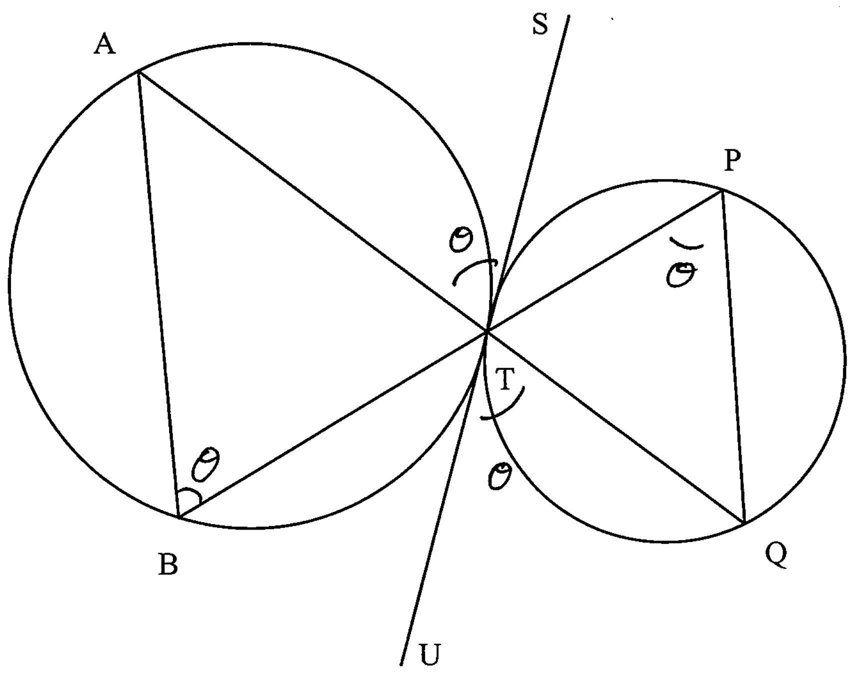


△ABD ≅ △CDB (AA)

4. (5 marks)

STU is a common tangent to both circles. AQ and BP are straight lines.

Prove that AB is parallel to PQ



Let $\angle ABT = \theta$

$\therefore \angle STA = \theta$ (Alternate segment)

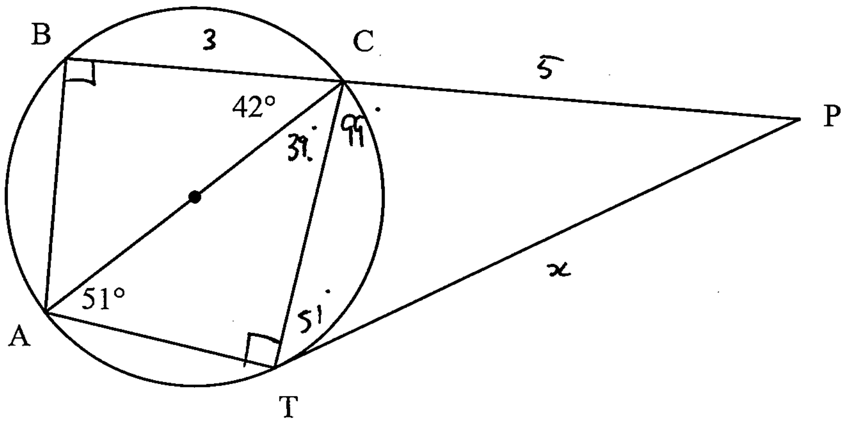
$\therefore \angle QTU = \theta$ (Vertically opposite)

$\therefore \angle TPQ = \theta$ (Alternate segment)

$\therefore AB \parallel PQ$ (Alternate angles are equal)

5. (7 marks)

Consider the following diagram. PT is a tangent to the circle.



Determine, with reasons

(a) $\angle ABC$ $= 90°$ (Angle is semi circle) [2]

(b) $\angle CPT$ [3]

$\angle ACT = 39°$ (Δ angle sum)

$\angle PCT = 99°$ (Supplementary angles)

$\angle CTP = 51°$ (Alternate segment)

$\therefore \angle CPT = 30°$ (Δ angle sum)

(c) $|PT|$ if $|BC| = 3$ and $|CP| = 5$ [2]

$(PT)^2 = |BP| \times |CP|$ (Chord Secant)

$= 8 \times 5$

$= 40$

$\therefore |PT| = \sqrt{40}$

$= 2\sqrt{10}$

TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 2/3 2020

Section 2 Calculator Assumed
Component Vectors & Geometric Proof

STUDENT'S NAME ______________________________

DATE: Wednesday 13 May **TIME:** 25 minutes **MARKS:** 24

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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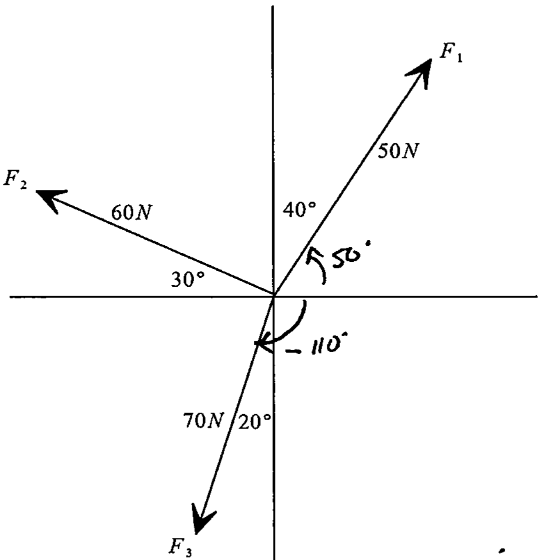
6. (5 marks)

Three forces act on a body as shown in the diagram below. Determine the magnitude and direction of a single force that will keep the system in equilibrium.



$$\underset{\sim}{R} = \begin{pmatrix} 50 \\ \angle 50° \end{pmatrix} + \begin{pmatrix} 60 \\ \angle 150° \end{pmatrix} + \begin{pmatrix} 70 \\ \angle -110° \end{pmatrix}$$

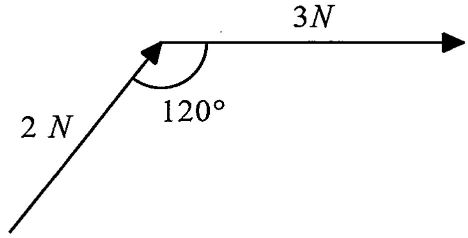
$$= \begin{pmatrix} -43.76 \\ 2.52 \end{pmatrix}$$

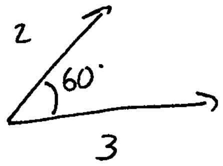
$$\therefore \underset{\sim}{E} = \begin{pmatrix} 43.76 \\ -2.52 \end{pmatrix} = \begin{pmatrix} 43.84 \\ \angle -3.3° \end{pmatrix}$$

$\therefore$ Equilibrium force is 43.84 N @ 093°T

7. (7 marks)

(a) Determine the scalar product for the two vectors shown. [2]





$2 \times 3 \times \cos 60°$

$= 2 \times 3 \times \frac{1}{2}$

$= 3$

(b) For the vectors $\mathbf{a} = -3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = 5\mathbf{i} + 2\mathbf{j}$ determine

(i) The vector projection of **a** onto **b** [3]

$$\text{proj}_{\underset{\sim}{b}} \underset{\sim}{a} = (a \cdot \hat{b})\,\hat{b} \qquad |\underset{\sim}{b}| = \sqrt{29}$$

$$= \frac{1}{29}\left(\begin{pmatrix}-3\\4\end{pmatrix} \cdot \begin{pmatrix}5\\2\end{pmatrix}\right)\begin{pmatrix}5\\2\end{pmatrix}$$

$$= \frac{-7}{29}\begin{pmatrix}5\\2\end{pmatrix}$$

(ii) The scalar projection of **b** onto **a** [2]

$$\text{proj}_{\underset{\sim}{a}} \underset{\sim}{b} = (\underset{\sim}{b} \cdot \hat{a})\,\hat{a}$$

$$= \underset{\sim}{b} \cdot \hat{\underset{\sim}{a}}$$

$$= \frac{1}{5}\begin{pmatrix}5\\2\end{pmatrix} \cdot \begin{pmatrix}-3\\4\end{pmatrix}$$

$$= \frac{1}{5}(-15 + 8)$$

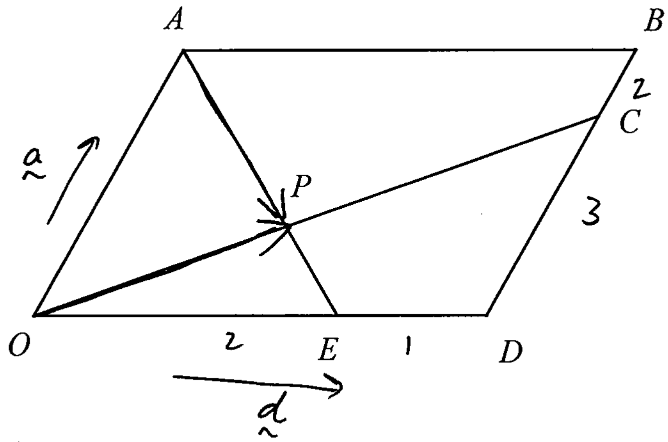
$$= \frac{-7}{5}$$

8. (6 marks)

Parallelogram $OABD$ has C on $\overrightarrow{DB}$ such that $\overrightarrow{DC} = \frac{3}{5}\overrightarrow{DB}$ and E on $\overrightarrow{OD}$ such that $\overrightarrow{OE} = \frac{2}{3}\overrightarrow{OD}$.

Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OD} = \underset{\sim}{d}$, $\overrightarrow{OP} = h\overrightarrow{OC}$ and $\overrightarrow{AP} = k\overrightarrow{AE}$ where P is the point of intersection of $\overrightarrow{AE}$ and $\overrightarrow{OC}$.

Determine the values of h and k.



$\vec{OP} = \vec{OA} + \vec{AP}$

$h\vec{OC} = \underset{\sim}{a} + k\vec{AE}$

$h(\underset{\sim}{d} + \frac{3}{5}\underset{\sim}{a}) = \underset{\sim}{a} + k(-\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{d})$

$\frac{3}{5}h\underset{\sim}{a} + h\underset{\sim}{d} = \underset{\sim}{a} - k\underset{\sim}{a} + \frac{2}{3}k\underset{\sim}{d}$

$\frac{3}{5}h\underset{\sim}{a} - \underset{\sim}{a} + k\underset{\sim}{a} = \frac{2}{3}k\underset{\sim}{d} - h\underset{\sim}{d}$

$\Rightarrow \quad \frac{3}{5}h - 1 + k = 0 \quad -①$

$\frac{2}{3}k - h = 0 \quad -② \quad \Rightarrow h = \frac{2}{3}k$

Sub ② into ①

$\frac{\cancel{3}}{5}\left(\frac{2}{\cancel{3}}k\right) - 1 + k = 0$

$2k - 5 + 5k = 0$

$k = 5/7$

$h = \frac{2}{3} \cdot \frac{5}{7}$

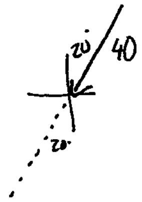
$h = \frac{10}{21}$

9. (6 marks)

Paul's aircraft can fly at 250 km/h in still air. It is to be flown from Suva in Fiji to his island getaway Presser Island, 300 km from Suva on a bearing 310°. There is a wind of 40 km/h blowing from 020°. Determine

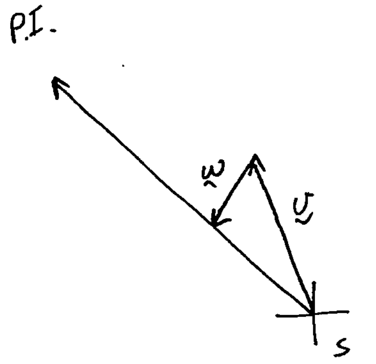
(a) the course Paul must set to fly directly to Presser Island [4]

wind



let $\underset{\sim}{v} = \begin{pmatrix} a \\ b \end{pmatrix}$

$$\underset{\sim}{w} = \begin{pmatrix} 40 \\ \angle -110^\circ \end{pmatrix} = \begin{pmatrix} -13.68 \\ -37.59 \end{pmatrix}$$



Now $\underset{\sim}{v} + \underset{\sim}{w} = \lambda(S\,P.I.)$

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} -13.68 \\ -37.59 \end{pmatrix} = \lambda \begin{pmatrix} -229.81 \\ 192.84 \end{pmatrix}$$

$\therefore$ we have

$$a - 13.68 = -229.81\lambda \quad -(1)$$
$$b - 37.59 = 192.84\lambda \quad -(2)$$
$$a^2 + b^2 = 250^2 \quad -(3)$$

Sub (1) & (2) into (3)

$$(13.68 - 229.81\lambda)^2 + (37.59 + 192.84\lambda)^2 = 250^2$$

$\lambda = 0.78$ $\quad \therefore \underset{\sim}{v} = \begin{pmatrix} -165.11 \\ +187.61 \end{pmatrix}$

$= 249.9$ km/h @ $319°$T

(b) the time taken for the flight (to the nearest minute). [2]

velocity actual = scale factor of distance

$\underset{\sim}{v} + \underset{\sim}{w} = \lambda\,(S\,P.I.)$

Units $\left[\frac{km}{h}\right] = \lambda\,[km] \implies \lambda = \left[\frac{1}{h}\right]$

$\therefore$ time $= \frac{1}{\lambda}$

$= 1.28$ hrs

or 1h 17 min